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The scale of market quakes

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1. Introduction

Prices in financial markets evolve as events occur. Events are typically market transactions, which may be correlated with each other, or news and political announcements. The price evolution occurs in many different forms and is difficult to describe concisely and systematically as price moves happen at all different price and time scales. For instance, a 1% price move can occur within a few seconds and the price jumps to its new level within a few minutes, and the price is subject to a secondary counter price move or, alternatively, for a few days the price may zig-zag within a narrow price range. These examples highlight the importance of creating a tool coming up with a concise abstraction that characterises the price evolution in a systematic manner so as to be a reliable representation of the state of the market at any point in time and after any type of market event, for example (Maillet and Michel 2005, Boucher et al. 2009). The tool would allow economic agents, be they governments or private and institutional investors, to take more informed decisions and adopt preemptive action in a crisis.

The pioneering work of Zumbach et al. (2000) defined, for the currency market, the so-called scale of market shocks that quantifies market movements on a tick-by-tick basis as a weighted average of volatilities over different time horizons. Zumbach et al. (2000) use the scale of market shocks to measure the impact of major events between 1998 and 1999 for a couple of exchange rates. This approach, certainly interesting, mainly suffers from being too complicated as too many ad-hoc choices of functions and fittings are made. Also, we note the arbitrary weighting of time horizons that can make the scale of market shocks to over- or, on the contrary, underweight a given time-scale, therefore distorting the measurement.

Maillet and Michel (2003) adapted the scale of market shocks to the stock market. The new indicator is designed for the detection and comparison of the severity of different crises and suffers from the same deficiencies as its ancestor. It is also worth mentioning the unpublished work of Subbotin (2008), which, also inspired by Zumbach et al. (2000), proposed a probabilistic indicator.
for volatilities by decomposing the volatility of stock market indexes using wavelets. Wavelet decomposition is used to circumvent the mixing of scales in Zumbach et al. (2000). Based on the results presented by Subbotin (2008), one is tempted to conclude that this indicator seems to be more suited for detecting crises or regime shifts rather than quantifying the impact of single events as we observe periods up to a year over which the indicator has a significant value.

For the choice of a metric to measure market evolution there cannot be a right or wrong. There are, however, two criteria for such a metric that seem particularly important: simplicity and ability to incorporate all the details of the price evolution of the time series. We claim that the previous attempts presented above have not been able to maximise these criteria. Zumbach et al. (2000), Maillot and Michel (2003) and Subbotin (2008) chose the volatility of the market to characterise its state. Although this sounds like a natural choice, we argue that aggregating market activity into a volatility measurement might not be the most appropriate to capture market activity that can take various forms (see figure 3) where the different activated price scales $\lambda$ (see below for a formal definition) are mingled. As we shall see below, computing market activity within our event-based approach appears to be more natural and effective. In addition, we stress that only the high-frequency definition of volatility in Zumbach et al. (2000) prevents information being lost through homogenising the time series, and, even more important, every time increment has the same weight.

Inspired by the discovery of a large number of scaling laws (Glattfelder et al. 2010) and the seminal work of Guillaume et al. (1997), we propose a framework in which sequences of price directional changes set the rhythm and where we monitor the excess price moves from one directional change to the next, the so-called overshoot. We note that dissecting the time series of prices according to price thresholds has also been considered by Simonsen et al. (2002), who introduced the so-called investment horizon concept that characterises the smallest time interval needed for an asset to cross a fixed return level. However, to the best of our knowledge, decomposing the time series of prices into directional changes and overshoot regions that, as we shall see below, allows us to quantify market activity, has not been published before.

Within our novel and simple framework, physical time no longer exists and is replaced by intrinsic time, ticking at every occurrence of a directional change of price. This framework is therefore ideally suited to deal with tick-by-tick data as it is not constrained by considering a fixed time scale. Within this framework, we design a methodology to quantify the impact of multi-scale events along a scale, the so-called scale of market quakes (SMQ), which defines a tick-by-tick metric allowing us to quantify market evolution on a continuous basis by analysing the average overshoot over representative price scales. The proposed bottom-up approach does not exhibit the drawbacks of volatility because, as we shall see, market activity is computed by aggregating the activity of the representative price scales allowing for an effective quantification of market activity. Without loss of generality, we apply our methodology to the FX market and publish real-time magnitudes online (www. olsenscale.com).

We are not primarily interested in designing a forecasting tool, but rather to develop, in the same vein as the Richter scale (Richter 1958) that assesses the magnitude of earthquakes, a tool that systematically quantifies market events once they have actually happened. However, further analysis of the outputs allows one to assess the volatility regime and its evolution, in turn helping to assess risk.

Related literature in the field explores how transactions impact the market (see Bouchaud et al. (2008) and references therein). In contrast, we are interested in developing a tool to quantify, in an objective manner, the trajectory of market prices evolution.

The document is organised as follows. We first describe the methodology defining the SMQ and show various news announcement snapshots to highlight the usefulness of the scale. We then analyse the evolution of magnitudes along the SMQ over the years across major currency pairs. Finally, we conclude and discuss further work.

2. Methodology

2.1. An event discretisation of the price curve

It is customary to discretise the price curve along its temporal axis by computing the return $r$ as the price difference within a time interval $\Delta t$, 

$$ r(t) = \log p(t) - \log p(t - \Delta t), \quad (1) $$

where $p(t)$ is the homogeneous, and therefore interpolated, sequence of prices. Volatility and other statistical analyses are computed from the time series (1).

Because market prices change at irregular time intervals, measurement of market activity in terms of discrete $\Delta t$ needs to be adaptive. To achieve that we propose an event-based approach that considers the sequence of price directional changes of magnitude $\lambda$ (Guillaume et al. 1997; Dacorogna et al. 2001, Glattfelder et al. 2010). Within that framework, time passes by unevenly: any occurrence of a directional change represents a new intrinsic time unit.

The dissection algorithm measures the occurrences of a price change $\lambda$ from the last high or low (i.e. an extremum), if it is in an up or down mode, respectively. At each occurrence of a directional change, the overshoot associated with the previous directional change is determined as the difference between the price level at which the last directional change occurred and the extremum. The high and low price levels are then reset to the current price and the mode alternates. Figure 1 shows how the price curve is dissected into directional change and overshoot sections and a pseudo-code in appendix A gives further details on how to dissect the time series of prices and also on how to determine the evolution of price overshoots.
A directional change of magnitude \( \lambda \) is usually not immediately followed by an opposite directional change, but rather by a price overshoot \( \hat{\omega}(t, \lambda) \) where \( t \) denotes physical time. The price overshoot is of particular interest as it measures the excess price move along a scale and can be used as a market activity quantifier.

The dynamics of overshoots organises as follows. Every occurrence of a directional change triggers a new overshoot that will swing between \( \lambda \) and any positive value until it decreases by \( -\lambda \), making the next directional change occur. Figure 2(a) shows the overshoot dynamics.

In order to capture the market activity that is not occurring at a single price scale we define an average overshoot \( \bar{\omega}(t) \) as

\[
\bar{\omega}(t) = \frac{1}{n} \sum_{i=1}^{n} \omega^q(t, \lambda_i),
\]

where \( n \) is the number of thresholds \( \lambda_i \) and the superscript \( q \) on \( \omega^q(t) \) denotes the fact that the overshoot \( \omega(t) \) is expressed in quantiles where the tick-by-tick historical distribution of price overshoots from December 1, 2005 up to December 31, 2008 is considered. Overshoots are normalised using quantiles so as to be averaged over different thresholds. We consider evenly distributed thresholds and set \( \lambda_i = i \cdot 0.05\% \) with \( i \) running from 1 to \( n = 100 \). For simplicity, we have chosen to equally weigh the various quantiles in the
average overshoot \( \tilde{\omega} \) that appears below to behave properly. We leave to further research the exploration of the effects of using alternative weightings. It is worth noting at this stage that both \( \tilde{\omega}(t) \) and \( \tilde{\omega}(t) \) are inhomogeneous. Figure 2(b) shows the time evolution of \( \tilde{\omega} \).

2.2. The scale of market quakes

Here we propose a way to convert the average overshoot \( \tilde{\omega}(t) \) into a unique number \( S(t) \) expressing the level of excitation of the market along a scale, the so-called scale of market quakes.

Day-to-day experience tells us that when the market is highly excited, price overshoots are both frequently reset (i.e. frequent directional changes) and greatly in excess of the originating directional changes. This means that, over a time window, the more hectic the market, the more the frequencies composing the signal \( \tilde{\omega}(t) \). To capture the market activity we therefore compute the Fourier spectrum of the signal \( \tilde{\omega}(t) \) over a time window and the magnitude \( S(t) \) is defined as a weighted average of the Fourier frequencies. A weighted average is considered in order to minimise the impact of high-frequency fluctuations and therefore ensures some robustness. Indeed, a small perturbation to any signal most probably implies very similar low Fourier frequencies, but significantly alters the high ones.

More specifically, a magnitude \( S(t) \) along the scale of market quakes is defined as

\[
S(t) = \frac{1}{n_a + 1} \sum_{i=0}^{n_a} \mathcal{F}\left( \Omega(t + \left( \frac{i}{n_a} - 0.5 \right) \delta t) \right),
\]

where \( t \) is the physical time, \( \delta t = 2 \) hours is the time window, \( n_a = \delta t/\delta t_w \) with \( \delta t_w = 15 \) minutes and the set \( \Omega(t) = [\tilde{\omega}(t) - \tilde{\omega}(t)] \cdot t - \delta t/2 \leq t \leq t + \delta t/2 \). We associate the quantity \( S(t) \) with the middle of the time window to reflect the state of the market both before and after news occurring at time \( t \). The average operator \( \langle \cdot \rangle \) is taken over \([t - \delta t/2; t + \delta t/2]\) and is used to prevent high or low plateaux corresponding to significantly different frequencies. The operator \( \mathcal{F}(\cdot) \) is defined as

\[
\mathcal{F}(\Omega(t)) = \frac{1}{n_f} \sum_{k=0}^{n_f-1} |X_k| \delta t/k + 1,
\]

where \( n_f = \delta t/\delta t_f \) is the number of discretisation points of \( \Omega(t) \) and \( |X_k| \) is the magnitude of the Fourier frequency computed from the discretised \( \Omega(t) \). The discretisation \( \delta t_f = 7 \) seconds is chosen to be fine enough to capture details of \( \Omega(t) \) but computationally doable in real time. It is set to be as near as possible to the power of 2 required for an efficient computation of the Fourier transform: \( n_f = 2^{10} = 1024 = 1028 - 4 = 7200/7 - 4 \) where the four first (i.e. the four oldest) values are disregarded. As a discrete Fourier transform of a signal composed of real values obeys the symmetry \( X_k = X_{n_f-k} \), we set \( n_f = [n_f/2 + 1] = 513 \).

The SMQ methodology is associated with a time lag of \( \delta t = 2 \) hours as computing a magnitude along the SMQ at time \( t \) requires knowing the time series up to \( t + \delta t/2 \) for computing \( \mathcal{F}(\Omega(t + \delta t/2)) \) and with a further \( \delta t/2 \) for properly defining \( \Omega \). To alleviate this issue we compute preliminary values by reducing the averaging scope \( n_a \) of equation (3) and set \( n_a = \{0, 2, 4, 6\} \), providing estimates 60, 75, 90 and 105 minutes after the news, respectively. On average, we note that estimates are roughly 20% higher than the final values (see www.olsenscale.com).

Temporary values are considered to reduce the impatience that might build when waiting 2 hours to obtain the magnitude of an event. They are, however, not designed to improve the forecasting ability of the SMQ, which is a tool to quantify the state of the market and not to forecast its future behaviour, even though further analysis of the SMQ signal can, for example, provide hints on the volatility regime, in turn helping to assess market risk.

We observe from (3) that magnitudes along the SMQ are strictly positive. An upper bound is found by manipulating the discrete Fourier transform definition to find a frequency bound, and then injecting it into (3). After some simple algebra, one finds

\[
S(t) \leq 50 \sum_{k=0}^{n_f-1} \frac{1}{k + 1}.
\]

However, as we shall see, \( S(t) \) is usually smaller than 10 and equation (5) hardly reaches its limits as it corresponds to a theoretical case.

3. Results

We use the SMQ to analyse the market on a 24 hour basis and evaluate the performance of the proposed methodology at major news announcements in the FX market. Due to its event-based nature, we observe that the SMQ is able to quantify price moves exhibiting different forms. In addition, we show that, as expected, a quake is not necessarily related to a news announcement. Then we examine the evolution of SMQ magnitudes between January 2004 and August 2009 and across different currency pairs where the credit crisis significantly alters the evolution of every pair. We stress here that the SMQ methodology is also applicable to other asset classes.

3.1. Quakes at event time

Figure 3(a)–(h) show the behaviour of EUR–USD and the SMQ on the occasion of eight releases of non-farm employment numbers (Bureau of Labor Statistics www.bls.gov). The wide variety of market responses (a steep drop (f), the same price move amplitude as in (f) but happening within a longer time period (e), little reaction from the market (c), volatile market (g,h) or a drop immediately followed by a recovery (b,g)) is a characteristic of our methodology of computing a single number within the SMQ. As expected, we observe that the steep drop (f) is associated with a larger value than (e)
where the difference between the two scenarios is mainly the time for the price move to occur. Scenario (b), that could well go unnoticed as the original price level does not seem to be altered by the news announcement, is given a significant magnitude that is comparable to (e).

We also notice from figure 3(a), (b) and (d) that peaks of magnitude do not always coincide with the release time, as the market response can take a few hours to operate.

It is also interesting to remark that, similarly to earthquakes, after-quakes occur such as in (a) and (g), and have, in contrast to what is shown here, also been observed to be stronger than the original quake. A reason for this might be that the first market reaction could trigger further actions, producing in turn market impacts of greater magnitude.

Figure 3(i) shows the distribution of the magnitude of two sets of events versus the maximum price move that occurred within the next 12 hours following the events. The first events (black circles) are 27 non-farm employment change announcements between 2007 and 2009, and the second events (gray circles) are 4687 magnitude peaks observed between December 2005 and March 2009, where a magnitude peak $S(t)$ is defined as $S(t) > S(t + 2\delta_t)$. The 10% and 90% quantiles of the distribution are shown.

Figure 3. (a)-(h) Behaviour of EUR–USD (thin lines) and the SMQ (thick line). The announcement time is depicted by a dashed line and its date appears at the top left of the figure. (i) Distribution of the magnitude of two sets of events versus the maximum price move that occurred within the next 12 hours following the events. The first events (black circles) are 27 non-farm employment change announcements between 2007 and 2009, and the second events (gray circles) are 4687 magnitude peaks observed between December 2005 and March 2009, where a magnitude peak $S(t)$ is defined as $S(t) > S(t + 2\delta_t)$. The 10% and 90% quantiles of the distribution are shown.
occurred within the next 12 hours following the events. The first events considered are 27 non-farm employment change announcements between 2007 and 2009, and the second are 4687 magnitude peaks observed between December 2005 and March 2009 where a magnitude peak corresponds to a value $S(t)$ where $S(t) > S(t \pm 2\delta_t)$.

We observe a cone-like structure where large magnitudes do not correspond to any small price moves but where large price moves can be associated with small magnitudes. This is because a large magnitude necessarily implies that high price thresholds are activated but, on the other hand, a noticeable price move can occur as a jump in the market and therefore does not necessarily correspond to a large magnitude, but rather to a lack of liquidity (Joulin et al. 2008).

Figure 3(i) also highlights that a magnitude peak can be high and not correspond to any non-farm employment change announcements. In fact, we have observed that most of these peaks do not coincide with any known news announcements, indicating that shocks in the market not only occur at scheduled news announcement times, but can also be unexpected and endogenous.

Figure 4. Time evolution of magnitude likelihoods $l_{100}(t, k)$ from July 2003 to August 2009 across representative currency pairs. Nested curves related to different magnitude thresholds $k \in [1; 6]$ are shown.
For the sake of presentation, we show here only data related to EUR–USD and only considering US news. There are no obstacles to considering other currency pairs and news and we are, as we write, applying this methodology to 24 currency pairs and publishing magnitudes related to the main international news (www.olsenscale.com).

We compared the output of the SMQ with an equivalently simple methodology that computes the standard deviation of 5 minute price returns over a 4 and a 12 hour time window. Overall, we observe a pickier and less bounded signal where small price variations are amplified when the time window is short and where, on the contrary, some activity is unnoticed for a larger time window. This is because, on one hand, the standard deviation computed over a small time window cannot capture the full amplitude of the price move as it is possibly not contained in the time window. On the other hand, increasing the time window has the tendency to blur out details as the number of observations increases. In addition to its simplicity, our approach has the conceptual difference that the information captured within the 4 hour time window, associated with the definition of $S(t)$, may originate from a directional change dating way back compared with the time window. Moreover, we do not bound our approach by considering a fixed time frame to compute price returns and therefore grasp all details of the market as they happen. This short comparison does not replace a more detailed one that should properly compare the SMQ with its ancestors (Zumback et al. 2000, Maillet and Michel 2003, Subbotin 2008). We however leave this for future research.

### 3.2. Time evolution of the magnitude

We now analyse the time evolution of SMQ magnitudes and define the magnitude likelihood $l_n(t,k)$ at time $t$ as the probability of observing a magnitude to be larger than or equal to a threshold magnitude $k$, within the last $n_d$ days

$$l_n(t, k) = \frac{1}{n_p + 1} \sum_{i=0}^{n_p} \mathcal{N}(t - i \Delta t_d), \quad (6)$$

where $n_p = n_d/2 \Delta t_d$ is the number of computed magnitudes within the backward-looking time window of $n_d$ days, and where $\mathcal{N}(t) = 1$ if $S(t) \geq k$ and $\mathcal{N}(t) = 0$ otherwise. In the following analysis we arbitrarily set a medium-term time window of $n_d = 100$ days.

Figure 4 shows the evolution of the magnitude likelihood $l_n(t,k)$ for representative currency pairs between July 2003 and August 2009 where the magnitude threshold $k$ varies from 1 to 6. As expected from definition (6), we observe that the smaller the threshold $k$, the larger the likelihood as $l_n(t,k_1) \leq l_n(t,k_2)$ if $k_1 \leq k_2$.

Overall, figure 4 shows decreasing likelihoods from July 2003 up to the middle of 2007. Then the first FX market response to the credit crisis shows that likelihoods exhibit bumps that peak at the highest amplitude so far and vanish in the middle of 2008. These peaks appear in the first quarter of 2008, implying that the violent price moves that occurred in summer 2007 might be responsible for these local extrema. It then follows the second market response as we observe sharp likelihood increases at the end of 2008. This is followed by significant decreases over 2009 as we measure up to a factor 4 between the maximum magnitude and the latest in August 2009. Figure 4 then seems to indicate that activity calms down in 2009, without indicating anything about what could happen next. The forecasting ability of this approach is however of interest and will be subject to further communication.

Figure 4(a) depicts the dynamics of EUR–CHF, which seems to behave in a somewhat different way than the other currency pairs presented in figure 4. Indeed, magnitude likelihoods appear to be smaller than 5% for all magnitude thresholds $k$, up to 2008. This is in line with a recent study by Glattfelder et al. (2010), who present a new set of 12 scaling laws that behave in a different way within EUR–CHF than within any of the 13 analysed currency pairs. The reason for this might be that the ratio between volatility and spread (difference between bid and ask) is lower in EUR–CHF than in any other currency pair analysed here, making it less attractive to speculative traders, and therefore generating less activity.

The analysis of the evolution of likelihoods would be enhanced by considering the average of likelihoods expressed in quantiles over all thresholds. A large average value would indicate an increase in the likelihood at all scales, therefore highlighting the probable start of a turbulent phase. We leave this point to future study.

### 4. Conclusions

We have proposed a new and simple way of quantifying price behaviour by computing the magnitudes of quakes along the scale of market quakes. The alternative quantifier is simple as it does not imply any ad-hoc choices of functions and no fitting needs to be made. The SMQ acts on a tick-by-tick basis (www.olsenscale.com) and quantifies multi-scale events occurring in the market in response to news announcements or a mismatch of demand and supply. The SMQ is a metric to measure the impact of these events.

We believe that the SMQ is a first step towards a global information system (Olsen and Cookson 2009) that we urgently need in order to assess the state of the economy and its financial markets. The information system would allow economic agents, be they governments or private and institutional investors, to take more informed decisions and adopt preemptive action in a crisis.

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We thank J. B. Glattfelder for providing figure 1.
Appendix A: Dissecting the time series of prices: The algorithm

Algorithm A1 dissects the time series of prices $p(t)$ into directional changes of amplitude $\lambda_i$ and overshoot regions $\omega_i(t, \lambda_i)$. 

**Algorithm A1:** Dissect the price curve from time $t_0$ and measure overshoots with $\lambda_i \geq 0$ price threshold

**Require:** initialise variables ($p^{\text{ext}} = p(t_0)$, mode is arbitrarily set to up, $p_{dc} = 0$)

1: if mode is down then
2: if $p_{dc} \neq 0$ then
3: $\omega(t) = (p_{dc} - p(t))/\lambda_i$
4: end if
5: if $p(t) < p^{\text{ext}}$ then
6: $p^{\text{ext}} \leftarrow p(t)$
7: else if $(p(t) - p^{\text{ext}})/p^{\text{ext}} \geq \lambda_i$ then
8: $p^{\text{ext}} \leftarrow p(t)$
9: mode $\leftarrow$ up
10: $p_{dc} = p(t)$
11: end if
12: else if mode is up then
13: if $p_{dc} \neq 0$ then
14: $\omega(t) = (p(t) - p_{dc})/\lambda_i$
15: end if
16: if $p(t) > p^{\text{ext}}$ then
17: $p^{\text{ext}} \leftarrow p(t)$
18: else if $(p^{\text{ext}} - p(t))/p^{\text{ext}} \geq \lambda_i$ then
19: $p^{\text{ext}} \leftarrow p(t)$
20: mode $\leftarrow$ down
21: $p_{dc} = p(t)$
22: end if
23: end if

References


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